Word Embeddings

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Motivation

- How to represent words in a neural network?
- Possible solution: indicator vectors of length $|V|$ (vocabulary size).

$$\mathbf{w}^{(\text{the})} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix} \quad \mathbf{w}^{(\text{cat})} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ \vdots \end{bmatrix} \quad \mathbf{w}^{(\text{dog})} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \end{bmatrix}$$

- **Question:** Why is this a bad idea?
  - Parameter explosion ($|V|$ might be $> 1M$)
  - All word vectors are orthogonal to each other $\rightarrow$ no notion of word similarity

Motivation

- Learn one word vector $\mathbf{w}^{(i)} \in \mathbb{R}^D$ ("word embedding") per word i
- Typical dimensionality: $50 \leq D \leq 1000 \ll |V|$
- Embedding matrix: $\mathbf{W} \in \mathbb{R}^{|V| \times D}$
- **Question:** Advantages of using word vectors?
  - We can express similarities between words, e.g., with cosine similarity:
  $$\cos(\mathbf{w}^{(i)}, \mathbf{w}^{(j)}) = \frac{\mathbf{w}^{(i)T}\mathbf{w}^{(j)}}{\|\mathbf{w}^{(i)}\|_2 \cdot \|\mathbf{w}^{(j)}\|_2}$$
  - Since the embedding operation is a *lookup operation*, we only need to update the vectors that occur in a given training batch

Motivation

- Training from scratch: Initialize embedding matrix randomly and learn it during training phase
- $\rightarrow$ words that play similar roles w.r.t. task get similar embeddings
- e.g., from sentiment classification, we might expect $\mathbf{w}^{(\text{great})} \approx \mathbf{w}^{(\text{awesome})}$
- **Question:** What could be a problem at test time?
  - If training set is small, many words are unseen during training and therefore have random vectors
- We typically have more unlabelled than labelled data. Can we learn embeddings from the unlabelled data?

Motivation

- Distributional hypothesis: "a word is characterized by the company it keeps"' (Firth, 1957)
- Basic idea: learn similar vectors for words that occur in similar contexts
- GloVe, Word2Vec, FastText

Questions?

Recap: Language Models

- **Question:** What is a Language Model?
  - Function to assign probability to a sequence of words.
- **Question:** What is an n-gram language Model?
  - Markov assumption: probability of word only depends on no more than $n-1$ other (previous) words:

$$P(w_{[1]} \dots w_{[T]}) = \prod_{t=1}^{T} P(w_{[t]} | w_{[t-1]} \dots w_{[t-n+1]})$$

Word2Vec as a Bigram Language Model

- Words in our vocabulary are represented as two sets of vectors:
  - $\mathbf{w}^{(i)} \in \mathbb{R}^D$ if they are to be predicted
  - $\mathbf{v}^{(i)} \in \mathbb{R}^D$ if they are conditioned on as context
- Predict word i given previous word j:

$$P(i|j) = f(\mathbf{w}^{(i)}, \mathbf{v}^{(j)})$$

- **Question:** What is a possible function $f(\cdot)$?

A Simple Neural Network Bigram Language Model

- Softmax!

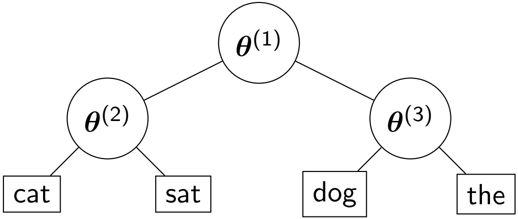
$$P(i|j) = \frac{exp(\mathbf{w}^{(i)T}\mathbf{v}^{(j)})}{\sum_{k=1}^{|V|} exp(\mathbf{w}^{(k)T}\mathbf{v}^{(j)})}$$

- **Question:** Problem with training softmax?
  - $\Rightarrow$ Slow. Needs to compute dot products with the whole vocabulary for every single prediction.

Questions?

Speeding up Training: Hierarchical Softmax

- Context vectors **v** are defined like before.
- Word vectors **w** are replaced by a binary tree:

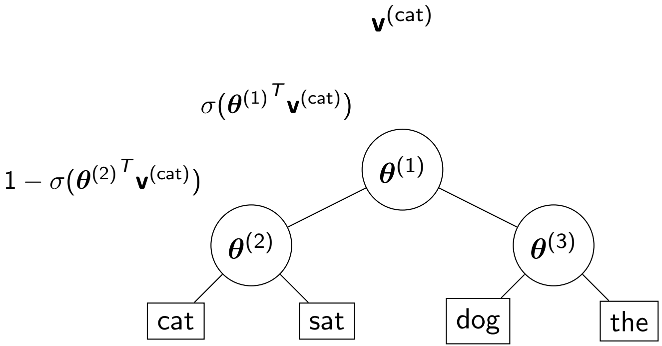


Hierarchical Softmax

- Each tree node l has parameter vector $\boldsymbol{\theta}^{(l)}$
- Probability of going left at node l given context word j:
  $p(\text{left}|l, j) = \sigma({\boldsymbol{\theta}^{(l)}}^T \mathbf{v}^{(j)})$
- Probability of going right: $p(\text{right}|l, j) = 1 - p(\text{left}|l, j)$
- Probability of word i given j: product of probabilities on the path from root to i

Example

Calculate $p(\text{sat}|\text{cat})$.



$$p(\text{sat}|\text{cat}) = \sigma(\boldsymbol{\theta}^{(1)}{}^T \mathbf{v}^{(\text{cat})})[1 - \sigma(\boldsymbol{\theta}^{(2)}{}^T \mathbf{v}^{(\text{cat})})]$$

Questions

- **Question:** How many dot products do we need to calculate to get to $p(i|j)$? How does this compare to the naive softmax?
  - $\log_2 |V| \ll |V|$
- **Question:** Show that $\sum_{i'} p(i'|j)$ sums to 1.

Questions?

Speeding up Training: Negative Sampling

- Another trick: **negative sampling** (aka *noise contrastive estimation*)
- This changes the objective function, and the resulting model is not a language model anymore!
- Idea: Instead of predicting probability distribution over whole vocabulary, make binary decisions for a small number of words.
- Positive training set: Bigrams seen in the corpus.
- Negative training set: Random bigrams (not seen in the corpus).

Negative Sampling: Likelihood

- Given:
  - Positive training set: $\mathrm{pos}(\mathcal{O})$
  - Negative training set: $\mathrm{neg}(\mathcal{O})$

$$L = \prod_{(i,j)\in \mathrm{pos}(\mathcal{O})} P(\mathrm{pos}|\mathbf{w}^{(i)}, \mathbf{v}^{(j)}) \prod_{(i',j')\in \mathrm{neg}(\mathcal{O})} P(\mathrm{neg}|\mathbf{w}^{(i')}, \mathbf{v}^{(j')})$$

- $P(\mathrm{pos}|\mathbf{w}, \mathbf{v}) = \sigma(\mathbf{w}^T\mathbf{v})$
- $P(\mathrm{neg}|\mathbf{w}, \mathbf{v}) = 1 - P(\mathrm{pos}|\mathbf{w}, \mathbf{v})$
- **Question:** Why not just maximize $\prod_{(i,j)\in \mathrm{pos}(\mathcal{O})} P(\mathrm{pos}|\mathbf{w}^{(i)}, \mathbf{v}^{(j)})$?
  - Trivial solution: make all $\mathbf{w}, \mathbf{v}$ identical

Word2Vec with negative sampling as classification

- Maximize likelihood of training data:

$$\mathcal{L}(\theta) = \prod_i P(y^{(i)}|x^{(i)};\theta)$$

- $\Leftrightarrow$ minimize negative log likelihood:

$$NLL(\theta) = -\log \mathcal{L}(\theta) = -\sum_i \log P(y^{(i)}|x^{(i)};\theta))$$

- **Question:** What do these components stand for in Word2Vec with negative sampling?
  - $x^{(i)}$ Word pair, from corpus OR randomly created
  - $y^{(i)}$ Label: $1 =$ word pair is from positive training set, $0 =$ word pair is from negative training set
  - θ Parameters $\mathbf{v}$, $\mathbf{w}$
  - $P(...)$ Logistic sigmoid: $P(1|\cdot) = \sigma(\mathbf{w}^T\mathbf{v})$, resp. $P(0|\cdot) = 1 - \sigma(\mathbf{w}^T\mathbf{v})$.

Stochastic Gradient Descent

$\frac{d\sigma(x)}{dx} = \sigma(x)(1-\sigma(x)) \qquad \frac{d\log(x)}{dx} = \frac{1}{x}$

$$L(\mathbf{w}, \mathbf{v}, y) = -y\log\big(\sigma(\mathbf{w}^T\mathbf{v})\big) - (1-y)\log\big(1-\sigma(\mathbf{w}^T\mathbf{v})\big)$$

$$\frac{\partial L}{\partial \mathbf{w}} =$$

$$\begin{aligned}
&- y\frac{1}{\sigma(\mathbf{w}^T\mathbf{v})}\sigma(\mathbf{w}^T\mathbf{v})\big(1-\sigma(\mathbf{w}^T\mathbf{v})\big)\mathbf{v} \\
&- (1-y)\frac{1}{1-\sigma(\mathbf{w}^T\mathbf{v})}(-1)\sigma(\mathbf{w}^T\mathbf{v})\big(1-\sigma(\mathbf{w}^T\mathbf{v})\big)\mathbf{v} \\
=&\big(\sigma(\mathbf{w}^T\mathbf{v}) - y\big)\mathbf{v}
\end{aligned}$$

Same for **v**:

$$\frac{\partial L}{\partial \mathbf{v}} = \big(\sigma(\mathbf{w}^T\mathbf{v}) - y\big)\mathbf{w}$$

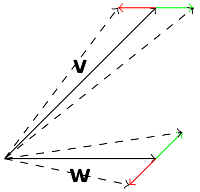
Stochastic Gradient Descent

- One update step for one word pair i, j:

$$\mathbf{v}^{(i)}_{updated} \leftarrow \mathbf{v}^{(i)} + \eta \left(y - \sigma(\mathbf{w}^{(i)T}\mathbf{v}^{(j)}) \right) \mathbf{w}^{(j)}$$

$$\mathbf{w}^{(j)}_{updated} \leftarrow \mathbf{w}^{(j)} + \eta \left(y - \sigma(\mathbf{w}^{(i)T}\mathbf{v}^{(j)}) \right) \mathbf{v}^{(i)}$$

- $\eta > 0$ is learning rate, y is label $\in \{0, 1\}$.
- When do the vectors of a pair become more/less similar, and why?
  - Let $c = \eta(y - \sigma(\mathbf{v}^{(i)T}\mathbf{w}^{(j)}))$
  - Positive (observed) word pair: $y = 1 \Longrightarrow c > 0$.
    - Hence, $c \cdot \mathbf{v}^{(i)}$ is added to $\mathbf{w}^{(j)}$ and vice versa $\rightarrow$ more similar.
  - Negative (random) word pair: $y = 0 \Longrightarrow c < 0$.
    - Hence, $c \cdot \mathbf{v}^{(i)}$ is subtracted from $\mathbf{w}^{(j)}$ and vice versa $\rightarrow$ less similar.



What does the step size c depend on?

Difference of y and $\sigma(\mathbf{w}^{(i)T}\mathbf{v}^{(j)})$

Speeding up Training: Negative Sampling

- Constructing a good negative training set can be difficult
- Often it is some random perturbation of the training data (e.g. replacing the second word of each bigram by a random word).
- The number of negative samples is often a multiple (1x to 20x) of the number of posisive samples
- Negative sets are often constructed per batch

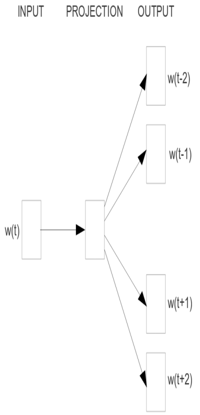
Questions

- **Question:** How many dot products do we need to calculate for a given word pair? How does this compare to the naive and hierarchical softmax?
    - $M + 1 \approx \log_2 |V| \ll |V|$
    - (for $M = 20, |V| = 1M$)

Questions?

Skip-gram (Word2Vec)

- Idea: Learn many bigram language models at the same time.
- Given word $w_{[t]}$, predict words inside a window around $w_{[t]}$:
  - One position before the target word: $p(w_{[t-1]}|w_{[t]})$
  - One position after the target word: $p(w_{[t+1]}|w_{[t]})$
  - Two positions before the target word: $p(w_{[t-2]}|w_{[t]})$
  - ... up to a specified window size c.
- Models share all $\mathbf{w}$, $\mathbf{v}$ parameters!



Skip-gram

Skip-gram: Objective

- Optimize the joint likelihood of the $2c$ language models:

$$p(w_{[t-c]} \dots w_{[t-1]} w_{[t+1]} \dots w_{[t+c]} | w_{[t]}) = \prod_{\substack{i \in \{-c \dots c\} \\ i \neq 0}} p(w_{[t+i]} | w_{[t]})$$

- Negative Log-likelihood for whole corpus (of size N):

$$NLL = -\sum_{t=1}^{N} \sum_{\substack{i \in \{-c \dots c\} \\ i \neq 0}} \log p(w_{[t+i]} | w_{[t]})$$

- Using negative sampling as approximation:

$$\approx -\sum_{t=1}^{N} \sum_{\substack{i \in \{-c \dots c\} \\ i \neq 0}} \log \sigma(\mathbf{w}_{[t+i]}^T \mathbf{v}_{[t]}) - \sum_{m=1}^{M} \sum_{t=1}^{N} \sum_{\substack{i \in \{-c \dots c\} \\ i \neq 0}} \log[1 - \sigma(\mathbf{w}_{[t+i]}^T \mathbf{v}^{(*)})]$$

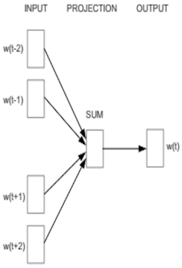
- $\mathbf{v}^{(*)}$ is a random context word, M is the number of negatives per positive sample

C(ontinuous) B(ag) o(f) W(ords)

- Like Skipgram, but...
- Predict word $w_{[t]}$, given the words inside the window around $w_{[t]}$:

$$p(w_{[t]} | w_{[t-c]} \dots w_{[t-1]} w_{[t+1]} \dots w_{[t+c]})$$

$$\propto \mathbf{w}_{[t]}^T \sum_{\substack{i \in -c \dots c \\ i \neq 0}} \mathbf{v}_{[t+i]}$$



CBOW

```
./word2vec -train data.txt -output vec.txt
        -window 5 -negative 20 -hs 0 -cbow 1
```

Questions?

FastText

- Even if we train Word2Vec on a very large corpus, we will still encounter unknown words at test time
- Orthography can often help us:
- $\mathbf{w}^{(\text{remuneration})}$ should be similar to
  - $\mathbf{w}^{(\text{remunerate})}$ (same stem)
  - $\mathbf{w}^{(\text{iteration})}, \mathbf{w}^{(\text{consideration})} \ldots$ (same suffix $\approx$ same POS)

FastText

$$\text{known word: } \mathbf{w}^{(i)} = \frac{1}{|\mathrm{ngrams}(i)| + 1} \Big[\mathbf{u}^{(i)} + \sum_{n \in \mathrm{ngrams}(i)} \mathbf{u}^{(n)}\Big]$$

$$\text{unknown word: } \mathbf{w}^{(i)} = \frac{1}{|\mathrm{ngrams}(i)|} \sum_{n \in \mathrm{ngrams}(i)} \mathbf{u}^{(n)}$$

$$\mathrm{ngrams}(\text{remuneration}) = \{\text{\$re}, \text{rem}, \text{\$rem}, \ldots \text{ration}, \text{ation\$}\}$$

FastText: Training

- ngrams typically contains 3- to 6-grams
- Replace $\mathbf{w}$ in Skipgram objective with its new definition
- During backpropagation, loss gradient vector $\frac{\partial J}{\partial \mathbf{w}^{(i)}}$ is distributed to word vector $\mathbf{u}^{(i)}$ and associated n-gram vectors $\mathbf{u}^{(n)}$

Summary

- Word2Vec as a bigram Language Model
- Hierarchical Softmax
- Negative Sampling
- Skipgram: Predict words in window given word in the middle
- CBOW: Predict word in the middle given words in window
- fastText: N-gram embeddings generalize to unseen words
- Any questions?

Initializing neural networks with pretrained embeddings

- Knowledge *transfer* from unlabelled corpus
- Design choice: Fine-tune embeddings on task or freeze them?
  - Pro: Can learn/strengthen features that are important for task
  - Contra: Training vocabulary is small subset of entire vocabulary → we might overfit and mess up topology w.r.t. unseen words

```
pretrained = #load_some_embeddings()
frozen = Embedding(input_dim = pretrained.shape[0],
        output_dim = pretrained.shape[1],
        weights = [pretrained],
        trainable = False)
finetunable = Embedding(input_dim = pretrained.shape[0],
        output_dim = pretrained.shape[1],
        weights = [pretrained],
        trainable = True)
```

(keras)

Initializing neural networks with pretrained embeddings

Model	MR	SST-1	SST-2	Subj	TREC	CR	MPQA
CNN-rand (randomly initialized)	76.1	45.0	82.7	89.6	91.2	79.8	83.4
CNN-static (pretrained+frozen)	81.0	45.5	86.8	93.0	92.8	84.7	**89.6**
CNN-non-static (pretrained+fine-tuned)	**81.5**	48.0	87.2	93.4	93.6	84.3	89.5
CNN-multichannel (combination)	81.1	47.4	**88.1**	93.2	92.2	**85.0**	89.4

Table from Kim 2014: Convolutional Neural Networks for Sentence Classification.

Resources

- `https://fasttext.cc/docs/en/crawl-vectors.html`
  - Embeddings for 157 languages, trained on big web crawls, up to 2M words per language
- `https://nlp.stanford.edu/projects/glove/`
  - GloVe word vectors: Cooccurrence-count objective, not n-gram based

Analogy mining

country-capital

$$\mathbf{w}^{(\text{Tokio})} - \mathbf{w}^{(\text{Japan})} + \mathbf{w}^{(\text{Poland})} \approx \mathbf{w}^{(\text{Warsaw})}$$

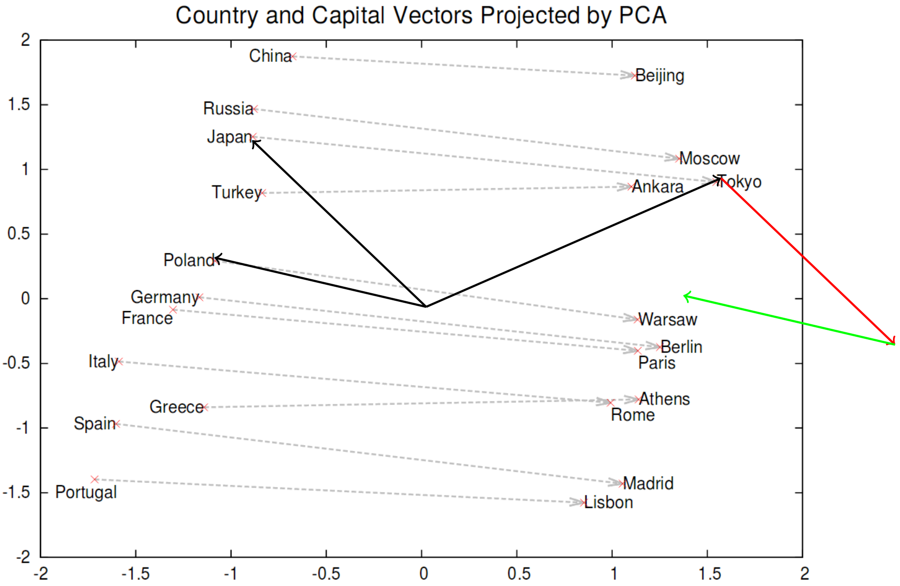
opposite

$$\mathbf{w}^{(\text{unacceptable})} - \mathbf{w}^{(\text{acceptable})} + \mathbf{w}^{(\text{logical})} \approx \mathbf{w}^{(\text{illogical})}$$

Nationality-adjective

$$\mathbf{w}^{(\text{Australian})} - \mathbf{w}^{(\text{Australia})} + \mathbf{w}^{(\text{Switzerland})} \approx \mathbf{w}^{(\text{Swiss})}$$

Analogy mining



$$\mathbf{w}^{(a)} - \mathbf{w}^{(b)} + \mathbf{w}^{(c)} = \mathbf{w}^{(?)}$$

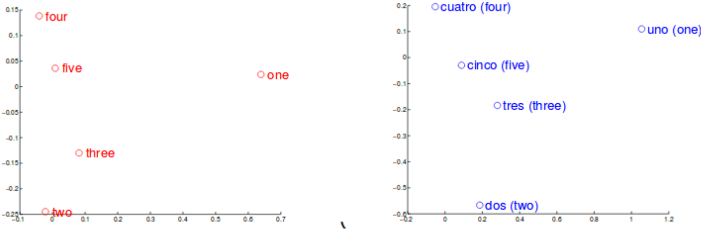
$$\mathbf{w}^{(d)} = \underset{\mathbf{w}^{(d')} \in \mathbf{W}}{\operatorname{argmax}} \quad \cos(\mathbf{w}^{(?)}, \mathbf{w}^{(d')})$$

Table 8: *Examples of the word pair relationships, using the best word vectors from Table 4 (Skip-gram model trained on 783M words with 300 dimensionality).*

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

Cross-lingual Embedding Spaces: A very short overview

- Embedding space: The space defined by the embeddings of all words in a language
- Hypothesis: Embedding spaces of different languages have similar structures



Mikolov et al. 2013: Exploiting Similarities among Languages for Machine Translation

Cross-lingual Embedding Spaces: A very short overview

- Given:
  - Monolingual embedding spaces of two languages: $\mathbf{W}_{L1}$, $\mathbf{W}_{L2}$
  - Dictionary D of a few known translations
- Learn function f, s.t.

$$\forall_{(i,j)\in D} f(\mathbf{w}_{L1}^{(i)}) \approx \mathbf{w}_{L2}^{(j)}$$

  - e.g., linear transformation: $f(\mathbf{w}_{L1}) = \mathbf{V}\mathbf{w}_{L1}$
- Given word k in L1 with unknown translation:
  - translate as L2 word l whose embedding $\mathbf{w}_{L2}^{(l)}$ minimizes cosine distance to $f(\mathbf{w}_{L1}^{(k)})$
- Used as initialization for unsupervised Machine Translation

Summary

- Applications of Word Embeddings:
- Word vector initialization in neural networks
- Analogy mining
- Word translation mining
- Any questions?